Small Signal Calculation of a SW RF Stage

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Our article "The Modern Armstrong Regenerative Receiver" presented a 530kHz~1700kHz MW regenerative detector based on the J310 N-channel JFET as a counterpart of the famous vacuum-tube homologous. The receiver there described was found to be very selective and sensitive. Tests conducted with different coil arrangements and an outside aerial suggested also excellent performance throughout the entire short wave spectrum.

When testing the receiver, the planetary reduction drive used in conjunction with the 475-pF broadcast variable proved to be very useful when tuning adjacent stations in the crowded SW bands. Although the author's SW prototype worked impressively well, some variations were devised thinking of experimenters wishing to replicate the receiver but lacking maybe space for a decent antenna. Duly attention was given for a minimum parts count.

There is the saying that a working receiver will be as good as its associated antenna-ground system. Situations exist where local conditions will render an outside random-wire antenna useless for sending good signals to a receiver. In this case, an RF-amplifier stage ahead of the regenerative detector will give the extra voltage gain required for a successful performance.

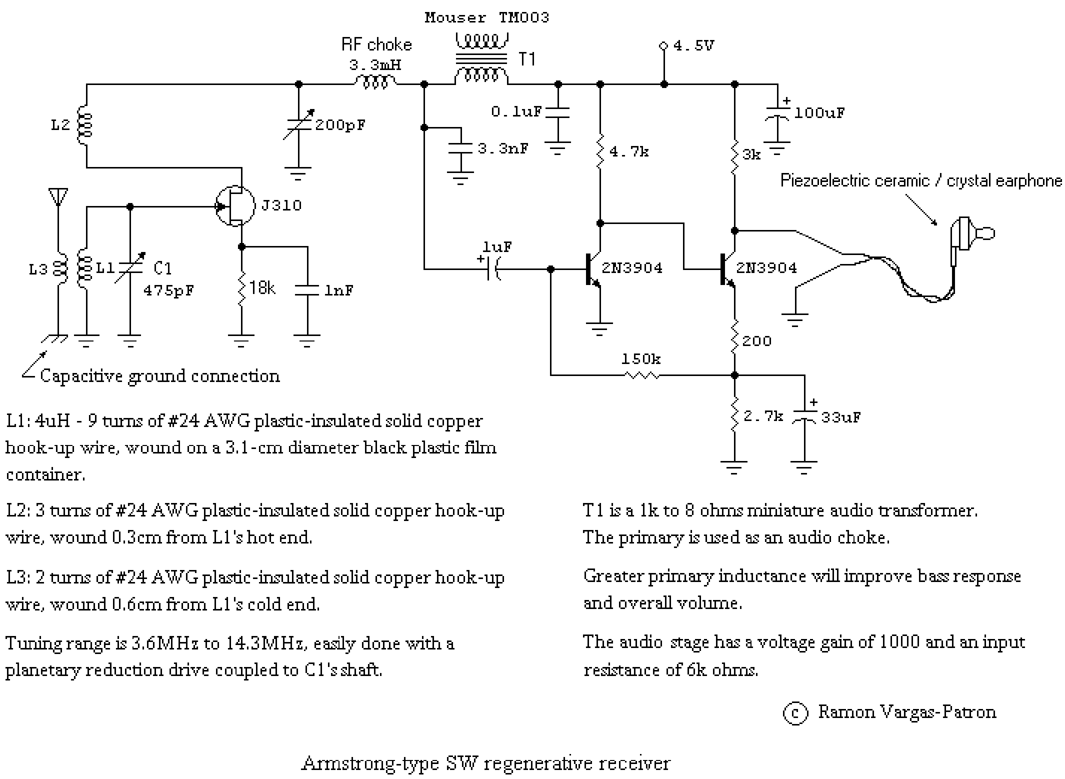


Fig.1 SW regenerative receiver

Fig.1 shows the schematic of the author's initial SW receiver prototype. Testing for better sensitivity, a very simple untuned RF amplifier using the MPF102 N-channel JFET in common-gate configuration was added to the J310-based SW receiver. This stage was capacitively coupled to the tuning tank of the regenerative detector. In order to take advantage of the maximum available voltage gain a 2.2-mH RF choke was connected between the JFET's drain electrode and the power supply. It was found that tight coupling to the detector stage would give the best noise / interference rejection. A low distributed-capacitance three-section pie-wound RF choke was selected for this application. Low-cost pile-wound chokes affected the upper end of the tuned band, precluding their use.

Fig.2 shows the schematic of the RF stage. The 10uH RF choke in series with the antenna lead-in effectively blocks FM / TV interference and a power grid-like annoyingly strong interference at the author's location. The series connected 100pF variable capacitor helps attenuating very strong signals and will also improve selectivity.

The output impedance of the RF stage depends on the impedances connected to its input. Having some gain control through the use of a variable input attenuator is advisable, given the dynamic range of available SW radio signals. However, this will bring about changes in the output impedance of the amplifier, which in turn will cause some receiver detuning.

In order to have some quantitative knowledge of the factors affecting the output impedance of the amplifier, the author conducted some calculations assuming operation in the mid band, so the device's parameters could be considered real quantities. For higher frequencies complex numbers would have to be used. However, the theoretical results obtained were coherent with the experimental observations at SW frequencies.

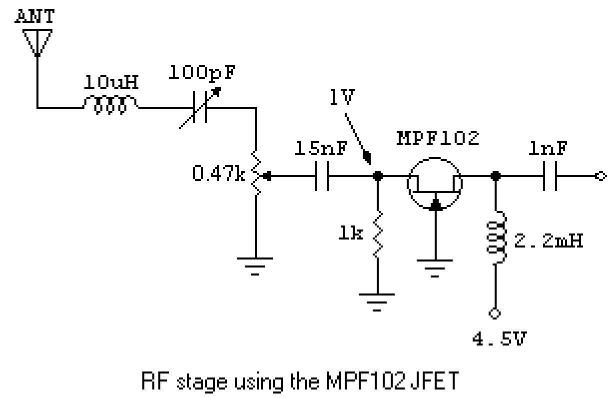


Fig.2 JFET-based RF stage

The small-signal models employed for gain and impedance calculations can be seen in Fig.3. Vg is the generator's signal voltage and Rg is the generator's output resistance. R_D is the drain's dynamic resistance and Rs is the source's bias external resistor. The JFET's small-signal model uses a vacuum tube-like circuit model. Hence, we need defining μ as the amplification factor, being $\mu = g_m R_D$. The quantity g_m is the JFET's forward transfer conductance for frequencies in the mid band.

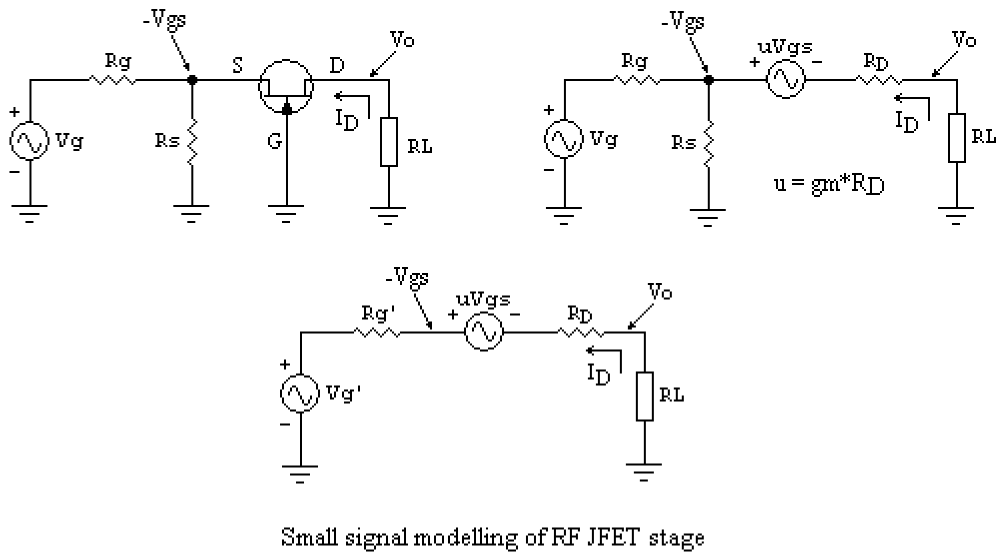


Fig.3 Small-signal models

In the figure above,

$$R_g' = R_g \,//\, R_s = \frac{R_g R_s}{R_g + R_s}$$

$$V_g' = V_g \frac{R_s}{R_s + R_g}$$

Solving for I_D gives:

$$-I_D = \frac{(\mu+1)V_g'}{(\mu+1)R_g' + R_D + R_L} = (\mu+1)V_g \frac{R_s}{R_s + R_g} \cdot \frac{1}{(\mu+1)R_g' + R_D + R_L}$$

Knowing that $V_o = I_o R_L$, where $I_o = -I_D$, we arrive at:

$$V_0 = (\mu+1)V_g \frac{R_s}{R_s + R_g} - I_0\left[(\mu+1)R_g' + R_D\right]$$

The output resistance is then:

$$R_0 = (\mu+1)R_g' + R_D = (\mu+1)(R_g \,//\, R_s) + R_D$$

Vo in terms of Vg yields an expression for the voltage gain Av = Vo / Vg:

$$A_v = \frac{V_0}{V_g} = \frac{(\mu+1)R_s R_L}{(R_s + R_g)\left[(\mu+1)(R_g // R_s) + R_D + R_L\right]}$$

which can also be written as:

$$A_v = \frac{(\mu+1)R_L}{\left(1+\frac{R_g}{R_s}\right)\left[(\mu+1)\frac{R_g}{\left(1+\frac{R_g}{R_s}\right)} + R_D + R_L\right]}$$

The input resistance of the stage as seen towards the JFET's source may be found to be:

$$R_{in} = \frac{V_g'}{(-I_D)} - R_g' = \frac{R_D + R_L}{(\mu+1)}$$

For the case the generator is an antenna, if we let Va be the voltage induced on the antenna by the radio waves, Rr the antenna-ground system's resistive losses plus the antenna's radiation resistance, Rp the variable input attenuator's total resistance and K the fraction of this resistance existing between the slider and the ground end, we get (Fig. 4):

$$V_g = V_a \frac{KR_p}{R_r + R_p}$$

$$R_g = \left[R_r + (1-K)R_p\right] // KR_p$$

The voltage gain from generator to load is :

$$Av_T = \frac{V_0}{V_a} = \frac{V_0}{V_g} \cdot \frac{V_g}{V_a} = \frac{KR_p}{R_r + R_p} A_v$$

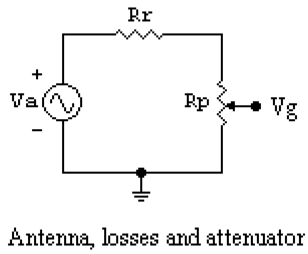


Fig.4 Antenna and input equivalent

Numerical examples

The following examples assume a JFET having the following small-signal mid-band parameters: g_m = 4mS, R_D = 40k ohms, $\mu = g_m R_D$ = 160. The antenna-ground system loss resistance plus the antenna's radiation resistance is Rr = 100 ohms. The source resistor is Rs = 1k ohms. The input attenuator's total resistance is Rp = 0.47k ohms and the output load's impedance is resistive and equal to R_L = 40k ohms.

K	$\frac{KR_p}{R_r + R_p}$	Rg	Av	Av_T	Ro
0.1	0.082	0.043kΩ	71.27	5.84	46.60kΩ
0.25	0.206	0.0933kΩ	62.84	12.94	53.68kΩ
0.5	0.412	0.138kΩ	56.86	23.43	59.48kΩ
0.75	0.618	0.134kΩ	57.35	35.44	59.02kΩ
1	0.824	0.082kΩ	64.55	53.19	52.24kΩ

Modifications of formulae for the bipolar case

The bipolar case (in a common-base configuration) requires that μ be made equal to g_m / h_{oe} and $1/h_{oe}$ substituted for R_D. Also, Re should be substituted for Rs . So we now have:

$$R_0 = (\mu + 1)(R_g // R_e) + \frac{1}{h_{oe}} \qquad , \mu = g_m / h_{oe}$$

The formula for Rg remains the same.

Explanation for why the receiver detunes when the gain control is adjusted

The tuning tank has series losses, mostly attributed to the coil, if the tuning capacitor is of a high-quality type. The lossy coil L can be converted into an equivalent ideal loss-less coil L' with a parallel loss r_p:

$$L' = L\left(1 + \frac{1}{Q_s^2}\right)$$

$$r_p = r_s\left(1 + Q_s^2\right)$$

where

$$Q_s = \frac{\omega_0 L}{r_s} = \frac{2\pi f_0 L}{r_s}$$

The coil L' now tunes with the capacitor to the same frequency ω_0 the lossy coil was tuning in conjunction with the cap:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L'C}}$$

When a coil has series losses, the tuned frequency is affected and will be given by a different formula. As can be seen L' is greater than L, so, given C, the lossy inductor L will tune to a lower frequency than that obtained with a lossless L.

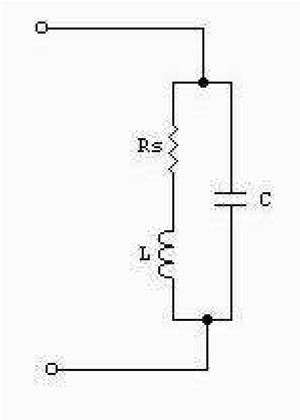
Now, the total parallel loss is r_{pt} = Ro // r_p. As has been seen, a manual change in the RF stage's voltage gain will cause variations in the output resistance Ro of the amplifier. Hence, the net parallel loss r_{pt} will change, driving us to accommodate regeneration values to the new situation. Regeneration partially cancels out parallel loss, which is a representation of the original series losses of the coil L. When cancellation is occurring, L' tends to the value L, so the tuned frequency changes. A new regeneration level implies a new value for L'.

Selecting a low value for Rg' = Rg // Rs will help reducing frequency detuning due to Ro variations. Bias issues usually impose constraints on the possible values for the source resistor Rs. So we are forced to minimize Rg, either by reducing Rr or selecting lower values for Rp. It´s much easier (and less expensive) to change the attenuator's total resistance than changing a complete antenna-ground system looking for a lower Rr.

APPENDIX

Type-I Lossy L-C Resonant Circuit

Consider an L-C parallel resonant circuit with series losses in the inductive branch.



The input admittance Y is given by:

$$Y = \frac{1}{R_s + j\omega L} + j\omega C \qquad \ldots(1)$$

$$= \frac{1 + j\omega C(R_s + j\omega L)}{R_s + j\omega L}$$

$$= \frac{1 - \omega^2 LC + j\omega R_s C}{R_s + j\omega L}$$

$$= \frac{(1 - \omega^2 LC + j\omega R_s C)(R_s - j\omega L)}{R_s^{\,2} + \omega^2 L^2}$$

$$= \frac{R_s + j\omega(\omega^2 L^2 C + R_s^{\,2} C - L)}{R_s^{\,2} + \omega^2 L^2} \qquad \ldots(2)$$

Resonance is attained when the admittance function´s phase is zero. This is, when:

$$\omega^2 L^2 C + R_s^{\,2} C - L = 0 \qquad \ldots(3)$$

The resonant frequency is given then by**:**

$$\omega_0^{\,2} = \frac{1}{LC}\left(1 - \frac{R_s^{\,2} C}{L}\right) < \frac{1}{LC} \qquad \text{...(4)}$$

The admittance at resonance is:

$$Y = \frac{R_s}{R_s^{\,2} + \omega_0^{\,2} L^2} \qquad \ldots(5)$$

The input impedance at resonance is Z = 1 / Y. Then:

$$Z = R_p = \frac{R_s^{\,2} + \omega_0^{\,2} L^2}{R_s}$$

$$= \frac{R_s^{\,2}\left(1 + Q_s^{\,2}\right)}{R_s}$$

$$= R_s\left(1 + Q_s^{\,2}\right) \qquad \ldots(6)$$

where:

$$Q_s = \frac{\omega_0 L}{R_s} \qquad \ldots(7)$$

is the inductor's Q factor at frequency ω_0.

Eq.(3) can be written as:

$$C\left(\omega_0^{\,2} L^2 + R_s^{\,2}\right) - L = 0$$

or

$$C R_s^{\,2}\left(Q_s^{\,2} + 1\right) - L = 0$$

Then,

$$\frac{R_s^{\,2} C}{L} = \frac{1}{Q_s^{\,2} + 1}$$

Substituting into Eq.(4) yields:

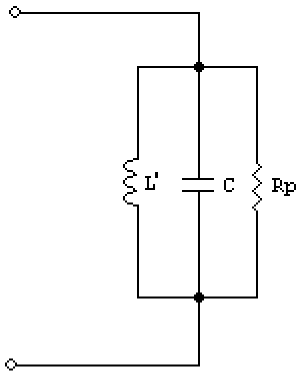
$$\omega_0^{\ 2} = \frac{1}{LC}\left(\frac{Q_s^{\ 2}}{Q_2^{\ 2}+1}\right)$$

$$\omega_0^{\ 2} = \frac{1}{\left(1+\frac{1}{Q_s^{\ 2}}\right)LC} \qquad \ldots(8)$$

The above expression suggests that parallel resonance is between capacitor C and an inductor

$$L' = L\left(1+\frac{1}{Q_s^{\ 2}}\right) \qquad \ldots(9)$$

Thus, the equivalent parallel resonant circuit is as shown in the figure below, for frequencies in the vicinity of ω_0.



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