DIRECTIONAL COUPLERS

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Abstract

This paper analyzes two types of Directional Couplers. First, magnetic coupling between a transmission line and a secondary circuit is studied. It is then shown that frequency independent samples of voltage and current on the transmission line yield a feasible way of obtaining separate readings of the forward and reflected waves.

The second type of coupler takes advantage of the electrostatic and magnetic coupling existing between two parallel conductors. A simplified model of the coupler permits a straightforward analysis of the circuit. Expressions for the forward and reflected voltages are then readily obtained. In both cases, a peak detector circuit gives DC readings of the voltages.

A directional coupler (D.C.) is a device that detects and separates the incident and reflected waves present in a transmission line, for instance, the one that links the radio transmitter with the antenna system.

One type of D.C. that makes use of voltage and current coupling is shown in Fig. 1, where it is suggested that the device be placed somewhere along the transmission line, between the signal generator (radio transmitter, for example) and the load Z_L (antenna). Usually, it is more comfortable to make the connection at the transmitter output.

Consider an unbalanced line of length ℓ, along with the circuit of Fig.1. If we call E_x the transmission line voltage in the connection point to the secondary circuit and I_x the current in the same point, we have:

$$\begin{aligned} E_x &= E_f\, e^{-j\,\omega x/\upsilon} + E_b\, e^{+j\,\omega x/\upsilon} \\ I_x &= \frac{E_f}{Z_0}\, e^{-j\,\omega x/\upsilon} - \frac{E_b}{Z_0}\, e^{+j\,\omega x/\upsilon} \end{aligned} \tag{1}$$

where:

E_f = incident component of voltage
E_b = reflected component of voltage
υ = velocity of propagation in the transmission line
Z_0 = characteristic impedance of the transmission line
x = position along the transmission line
ω = angular frequency of the generator

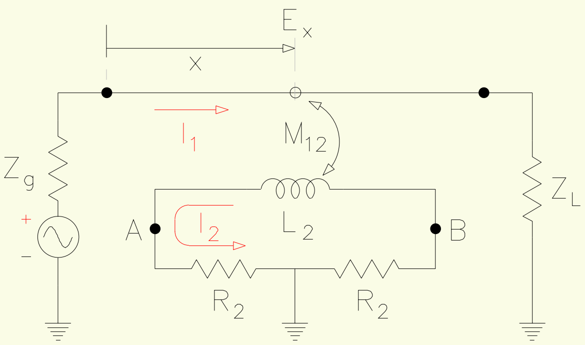


Figure 1. *Unbalanced line and secondary circuit*

We have for the secondary mesh, with $I_1 = I_x$:

$$j\,\omega\, M_{12}\, I_1 = \left(2R_2 + j\,\omega\, L_2\right) I_2 \tag{2}$$

Therefore:

$$\frac{I_2}{I_1} = \frac{j\,\omega\, M_{12}}{2R_2 + j\,\omega\, L_2} \tag{3}$$

If we make

$$\omega\, L_2 \gg 2R_2 \tag{4}$$

then the expression (3) becomes:

$$\frac{I_2}{I_1} = \frac{M_{12}}{L_2} \tag{5}$$

Notice that for this condition I_1 and I_2 are in phase and the frequency dependent term vanishes. Voltages in A and B will then be:

$$\begin{aligned} E_A &= \quad I_2\, R_2 = \quad R_2\, \frac{M_{12}}{L_2}\, I_1 \\ E_B &= -I_2\, R_2 = -\,R_2\, \frac{M_{12}}{L_2}\, I_1 \end{aligned} \tag{6}$$

Now, a frequency independent voltage sample may be obtained with the help of a capacitive divider, as is shown in Fig. 2. Then:

$$E_C = E_x\, \frac{C_1}{C_1 + C_2} \tag{7}$$

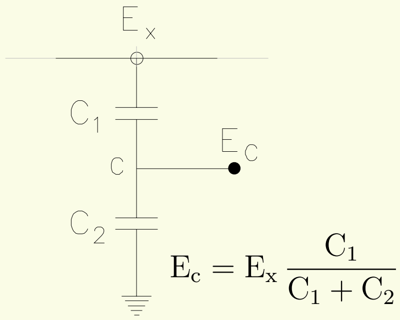


Figure 2. *Capacitive divider*

or

$$E_C \approx E_x \frac{C_1}{C_2} \tag{8}$$

if $C_2 \gg C_1$

The circuit for our coupler would become the one in Fig. 3. The following holds:

$$E_{AC} = E_A - E_C = R_2 \frac{M_{12}}{L_2} I_1 - E_x \frac{C_1}{C_2} \tag{9}$$

Substituting for E_x and I_1 the expressions labeled as (1), we have:

$$E_{AC} = R_2 \frac{M_{12}}{L_2} \left(\frac{E_f}{Z_0} e^{-j\,\omega x/\upsilon} - \frac{E_b}{Z_0} e^{+j\,\omega x/\upsilon} \right) - \frac{C_1}{C_2} \left(E_f\, e^{-j\,\omega x/\upsilon} + E_b\, e^{+j\,\omega x/\upsilon} \right) \tag{10}$$

If the following holds:

$$\frac{C_1}{C_2} = R_2 \frac{M_{12}}{L_2} \left(\frac{1}{Z_0} \right) \tag{11}$$

the terms containing E_f cancel each other and

$$E_{AC} = -2 \frac{C_1}{C_2} E_b\, e^{+j\,\omega x/\upsilon} \tag{12}$$

Also:

$$E_{BD} = E_B - E_D = -R_2 \frac{M_{12}}{L_2} I_1 - E_x \frac{C_1}{C_2} \tag{13}$$

Substituting for E_x and I_1 the expressions labeled as (1) and taking into account (11):

$$E_{BD} = -2 \frac{C_1}{C_2} E_f\, e^{-j\,\omega x/\upsilon} \tag{14}$$

We thus have a directional coupler with readings of the incident and reflected waves. Capacitor C_1 can be made adjustable for calibration purposes and to assure good directivity. If required, DC voltages can be obtained to drive a galvanometer, rectifying and filtering voltages E_{AC} for the reflected wave and E_{BD} for the incident component.

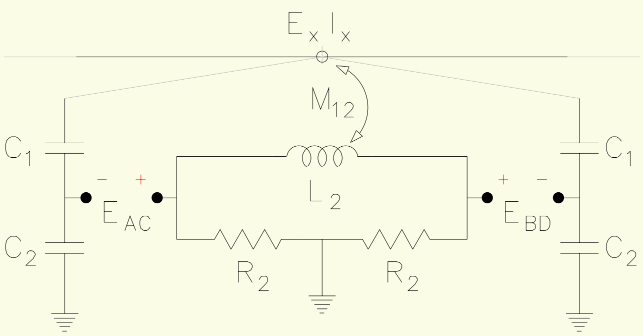
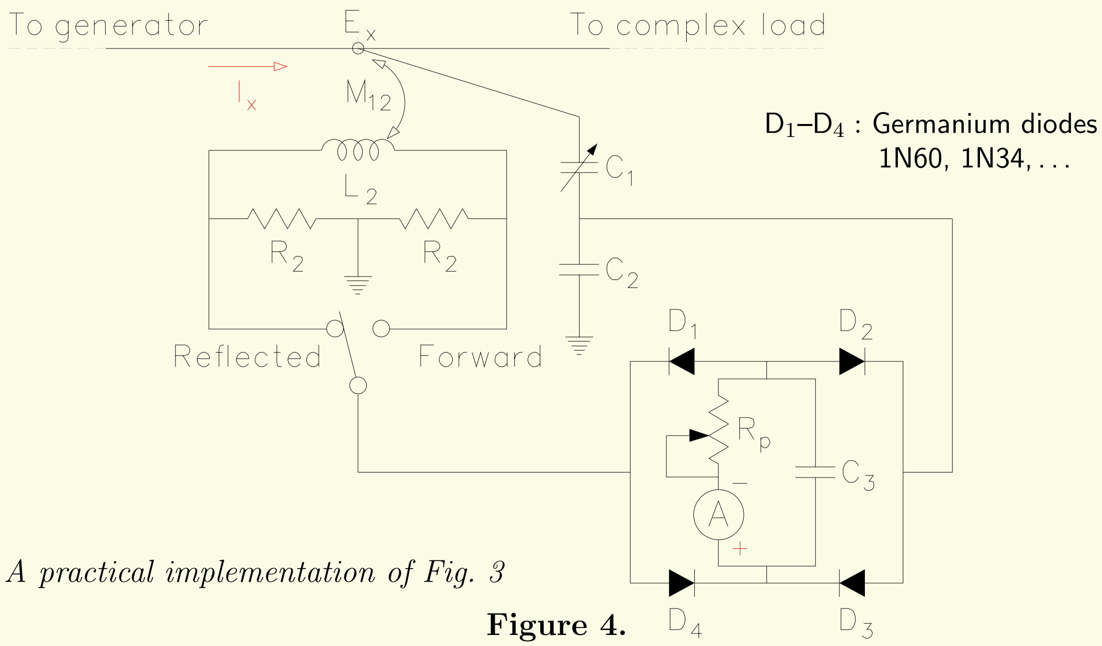


Figure 3. *Basic coupler circuit topology*

One possible implementation of this directional coupler is shown in Fig. 4 on following page.



A practical implementation of Fig. 3

Figure 4.

In Fig. 5 is shown a second type of D.C. that utilizes two parallel conductors with magnetic and electrostatic coupling. The main conductor is an extension of the transmission line that links the instrument with the generator at one end and with the load (antenna) at the other. The second conductor coupled to the previous one is end terminated with a resistive load and a detector circuit, respectively.

In these circuits readings from the incident or reflected waves can be selected by the switch.

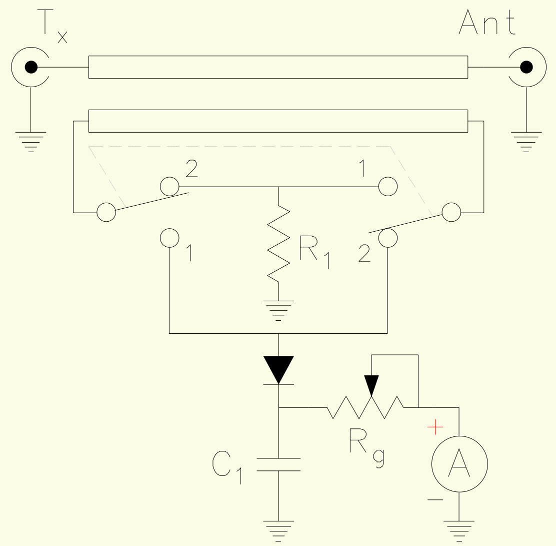


Figure 5. *A C-M type directional coupler*

A relatively simple analysis of the coupler can be made utilizing the equivalent circuit of Fig. 6. Here, C represents the distributed capacitance between conductors; M is the mutual inductance of the system; R_1 and R_2 are the end terminations of the secondary conductor and L is the self-inductance of this conductor. The transmission line current at the point where the device connects is I (complex quantity) and E is the voltage on the line at that same point.

The next calculations assume that the following inequality holds for R_1 (and also for R_2):

$$\omega\,L \ll R_1 \ll \frac{1}{\omega\,C} \tag{15}$$

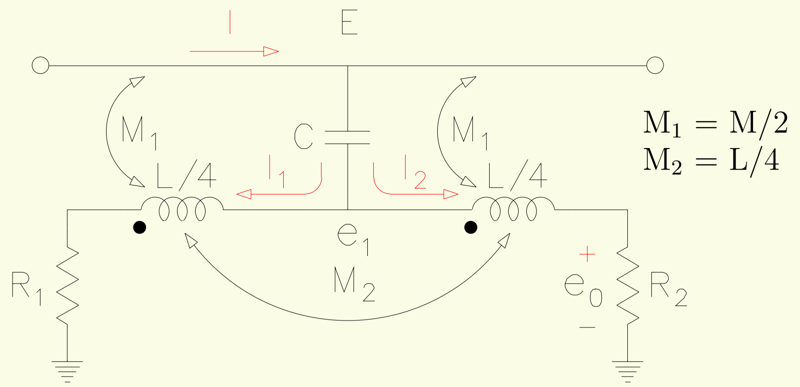


Figure 6. *The directional coupler's equivalent circuit*

In Fig. 6:

$$e_0 = -j\,\omega\,M_1 I - j\,\omega\,M_1 I + j\,\omega\,\frac{L}{4}\,I_1 - j\,\omega\,M_2 I_2 + j\,\omega\,M_2 I_1 - j\,\omega\,\frac{L}{4}\,I_2 + I_1 R_1 \tag{16}$$

Substituting for M1 and M2:

$$e_0 = -j\,\omega\,M\,I + j\,\omega\,\frac{L}{4}\,I_1 + j\,\omega\,\frac{L}{4}\,I_1 - j\,\omega\,\frac{L}{4}\,I_2 - j\,\omega\,\frac{L}{4}\,I_2 + I_1 R_1 \tag{17}$$

$$e_0 = -j\,\omega\,M\,I + j\,\omega\,\frac{L}{2}\,I_1 - j\,\omega\,\frac{L}{2}\,I_2 + I_1 R_1 \tag{18}$$

According to inequality (15), we can write:

$$e_0 \approx -j\,\omega\,M\,I + I_1 R_1 - j\,\omega\,\frac{L}{2}\,I_2 \tag{19}$$

On the other hand:

$$e_1 = -j\,\omega\,M_1 I + j\,\omega\,\frac{L}{4}\,I_1 - j\,\omega\,M_2 I_2 + I_1 R_1 \tag{20}$$

Also:

$$e_1 = +j\,\omega\,M_1 I + j\,\omega\,\frac{L}{4}\,I_2 - j\,\omega\,M_2 I_1 + I_2 R_2 \tag{21}$$

Equating (20) with (21):

$$\left(R_1 + j\,\omega\,\frac{L}{4}\right) I_1 - j\,\omega\,M_1 I - j\,\omega\,M_2 I_2 = \left(R_2 + j\,\omega\,\frac{L}{4}\right) I_2 + j\,\omega\,M_1 I - j\,\omega\,M_2 I_1 \tag{22}$$

$$\therefore \qquad \left(R_1 + j\,\omega\,\frac{L}{4} + j\,\omega\,M_2\right) I_1 = 2j\,\omega\,M_1 I + \left(R_2 + j\,\omega\,\frac{L}{4} + j\,\omega\,M_2\right) I_2 \tag{23}$$

or

$$\left(R_1 + j\,\omega\,\frac{L}{2}\right) I_1 = j\,\omega\,M\,I + \left(R_2 + j\,\omega\,\frac{L}{2}\right) I_2 \tag{24}$$

For the sake of inequality (15):

$$I_1 R_1 \approx j\,\omega\,M\,I + I_2 R_2 \tag{25}$$

Also:

$$\mathrm{I_1 + I_2 = (E - e_1)\, j\,\omega\, C} \tag{26}$$

with (20):

$$\mathrm{I_1 + I_2 = \left[E - \left(R_1 + j\,\omega\,\frac{L}{4}\right) I_1 + j\,\omega\,\frac{M}{2}\, I + j\,\omega\,\frac{L}{4}\, I_2\right] j\,\omega\, C} \tag{27}$$

or

$$\mathrm{I_1 + I_2 = \left(E + j\,\omega\,\frac{M}{2}\, I\right) j\,\omega\, C - I_1\left(R_1 + j\,\omega\,\frac{L}{4}\right) j\,\omega\, C + \left(j\,\omega\,\frac{L}{4}\, I_2\right) j\,\omega\, C} \tag{28}$$

From inequality (15) we get:

$$\mathrm{\omega^2\, LC \ll \omega\, R_1 C \ll 1} \tag{29}$$

Expression (26) becomes then:

$$\mathrm{I_1 + I_2 \approx \left(E + j\,\omega\,\frac{M}{2}\, I\right) j\,\omega\, C} \tag{30}$$

Solving for $\mathrm{I_1}$ from the last expression:

$$\mathrm{I_1 = \left(E + j\,\omega\,\frac{M}{2}\, I\right) j\,\omega\, C - I_2} \tag{31}$$

Substituting in (25):

$$\mathrm{\left[\left(E + j\,\omega\,\frac{M}{2}\, I\right) j\,\omega\, C - I_2\right] R_1 = j\,\omega\, M\, I + I_2 R_2} \tag{32}$$

$$\mathrm{j\,\omega\, CR_1\, E - j\,\omega\, M\, I\left(1 - \frac{j\,\omega\, CR_1}{2}\right) = I_2\,(R_1 + R_2)} \tag{33}$$

From (33) and according to (15):

$$\mathrm{I_2 \approx \frac{j\,\omega\, CR_1\, E - j\,\omega\, M\, I}{R_1 + R_2}} \tag{34}$$

On the other hand:

$$\mathrm{e_0 = I_2\, R_2} \tag{35}$$

(34) in (35):

$$\mathrm{e_0 = \frac{R_2}{R_2 + R_1}\,(-j\,\omega\, M\, I + j\,\omega\, CR_1\, E)} \tag{36}$$

If:

$$\mathrm{R_1 = R_2 = R \quad and \quad M = CR_1\, Z_0} \tag{37}$$

$$\mathrm{\therefore \qquad e_0 = \frac{1}{2}\, j\,\omega\, CR\,(E - Z_0 I)} \tag{38}$$

From the equations for a transmission line

$$\begin{aligned} \mathrm{E} &= \mathrm{E_f\, e^{-j\,\beta x} + E_b\, e^{+j\,\beta x}} \\ \mathrm{I} &= \mathrm{I_f\, e^{-j\,\beta x} - I_b\, e^{+j\,\beta x}} \end{aligned} \tag{39}$$

With $\beta = \omega/v$, and letting x = 0 (generator side)

$$\begin{aligned} \mathrm{E} &= \mathrm{E_f} + \mathrm{E_b} \\ \mathrm{I} &= \mathrm{I_f} - \mathrm{I_b} = \frac{\mathrm{E_f}}{\mathrm{Z_0}} - \frac{\mathrm{E_b}}{\mathrm{Z_0}} \end{aligned} \tag{40}$$

Then:

$$\mathrm{E} - \mathrm{Z_0 I} = 2\,\mathrm{E_b} \tag{41}$$

Consequently:

$$\mathrm{e_0} = \mathrm{j}\,\omega\,\mathrm{CR}\,\mathrm{E_b} \tag{42}$$

From (25), (37) and (42):

$$\mathrm{I_1\,R} = \mathrm{j}\,\omega\,\mathrm{CR}\,\mathrm{Z_0 I} + \frac{1}{2}\mathrm{j}\,\omega\,\mathrm{CR}\,(\mathrm{E} - \mathrm{Z_0 I}) \tag{43}$$

$$= \frac{1}{2}\mathrm{j}\,\omega\,\mathrm{CR}\,\mathrm{Z_0 I} + \frac{1}{2}\mathrm{j}\,\omega\,\mathrm{CR}\,\mathrm{E} \tag{44}$$

$$= \frac{1}{2}\mathrm{j}\,\omega\,\mathrm{CR}\,(\mathrm{E} + \mathrm{Z_0 I}) \tag{45}$$

and according to (40):

$$\mathrm{I_1\,R} = \frac{1}{2}\mathrm{j}\,\omega\,\mathrm{CR}\,(2\mathrm{E_f}) \tag{46}$$

$$= \mathrm{j}\,\omega\,\mathrm{CR}\,\mathrm{E_f} \tag{47}$$

Expressions (42) and (47) show us that we have separated the transmission line's incident and reflected waves.

Conclusions

Employing adequate circuit models two directional couplers have been studied. It has been shown that it is possible to obtain separate readings of the voltages for the incident and reflected components. This study helps the understanding of the principles of operation of actual devices in use at frequencies in the HF to UHF range.

References

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2 VARGAS PATRON, R., Lab Notes